1º) Calcula una primitiva $F(x)$ de la función $f(x)=\frac{4}{3\sqrt{2x-5}}$ tal que $F(3)=0$.

Resolución

La primitiva que buscamos estará en su integral indefinida:

$$F(x)=\int \frac{4}{3\sqrt{2x-5}}\,dx=\frac{2}{3}\cdot\int\frac{2}{\sqrt{2x-5}}\,dx=\frac{4}{3}\cdot\sqrt{2x-5}+c$$

$$F(3)=0 \Leftrightarrow \frac{4}{3}+c=0 \Leftrightarrow c=-\frac{4}{3}$$

La primitiva es $F(x)=\frac{4}{3}\cdot\left(\sqrt{2x-5}-1\right)$

2º) Calcula $\int_{-1}^{0}\frac{e^{-x}}{1+e^{-x}}\,dx$

Resolución

$$\int_{-1}^{0}\frac{e^{-x}}{1+e^{-x}}\,dx=-\int_{-1}^{0}\frac{-e^{-x}}{1+e^{-x}}\,dx=\left[-L(1+e^{-x})\right]_{-1}^{0}=-L2+L(1+e)=L\left(\frac{1+e}{2}\right)$$

3º) Calcula las siguientes integrales:

Resolución

a) $\int\frac{(x-1)^2}{\sqrt{x}}\,dx=\int x^2\cdot x^{-\frac{1}{2}}\,dx-2\int x\cdot x^{-\frac{1}{2}}\,dx+\int x^{-\frac{1}{2}}dx=\int x^{\frac{3}{2}}\,dx-2\int x^{\frac{1}{2}}\,dx+\int x^{-\frac{1}{2}}dx=$

$=\frac{x^{\frac{5}{2}}}{\frac{5}{2}}-2\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+c=\frac{2}{5}x^2\sqrt{x}-\frac{4}{3}x\sqrt{x}+2\sqrt{x}+c$

b) $\int\frac{x^2+5}{x^2+4}dx=\int\frac{x^2+4+1}{x^2+4}dx=\int xdx+\int\frac{1}{x^2+4}dx=x+\int\frac{1/4}{1+\left(\frac{x}{2}\right)^2}dx=x+\frac{1}{2}arctg\left(\frac{x}{2}\right)+c$

c) $\int\frac{2x-1}{x\cdot(x-1)\cdot(x+2)}\,dx$

Descomponemos el integrando en suma de fracciones simples:

$$\frac{2x-1}{x\cdot(x-1)\cdot(x+2)}=\frac{A}{x}+\frac{B}{x-1}+\frac{C}{x+2}=\frac{A\cdot(x-1)\cdot(x+2)+B\cdot x\cdot(x+2)+C\cdot x\cdot(x-1)}{x\cdot(x-1)\cdot(x+2)}$$

$$2x-1=A\cdot(x-1)\cdot(x+2)+B\cdot x\cdot(x+2)+C\cdot x\cdot(x-1)$$

Sustituyendo $x=0$; $x=1$ y $x=-2$ obtenemos $\begin{cases}-1=-2A\\ 1=3B\\ -5=6C\end{cases}$ de donde $A=\frac{1}{2}$; $B=\frac{1}{3}$; C=$\frac{-5}{6}$

Por tanto:

$\int\frac{2x-1}{x\cdot(x-1)\cdot(x+2)}\,dx\frac{1}{2}\int\frac{1}{x}\,dx+\frac{1}{3}\int\frac{1}{x-1}\,dx-\frac{5}{6}\int\frac{1}{x+2}\,dx=\frac{1}{2}L|x|+\frac{1}{3}L|x-1|-\frac{5}{6}L|x+2|$+c

d) $\int\frac{-2senx}{(2+cosx)^2}\,dx=2\cdot\int -senx\cdot(2+cosx)^{-2}\,dx=2\cdot\frac{(2+cosx)^{-1}}{-1}+c=\frac{-2}{2+cosx}+c$

e) $\int x\cdot(1-Lx)\,dx=\frac{x^2}{2}(1-Lx)+\frac{1}{2}\int xdx=\frac{x^2}{2}(1-Lx)+\frac{x^2}{4}+c=\frac{x^2}{2}\left(\frac{3}{2}-Lx\right)+c$

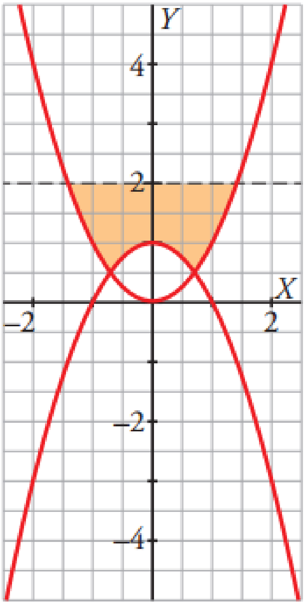
$$\begin{matrix}u=1-Lx\\ dv=x\,dx\end{matrix}\Longrightarrow\begin{matrix}du=-\frac{1}{x}dx\\ v=\int dv=\int x\,dx=\frac{x^2}{2}\end{matrix}$$

4º) Dibuja el recinto limitado por las gráficas de las siguientes funciones

$$y = x^2 \;\; ; \;\; y = 1 - x^2 \;\; ; \;\; y = 2$$

y calcula su área.

Resolución



Cortes $y = x^2$ e $y = 1 - x^2$:

$$x^2 = 1 - x^2 \Rightarrow 2x^2 = 1 \Rightarrow x = \pm\frac{\sqrt{2}}{2}$$

Cortes $y = x^2$ e $y = 2$:

$$x^2 = 2 \Rightarrow x = \pm\sqrt{2}$$

Cortes $y = 1 - x^2$ e $y = 2$:

$$1 - x^2 = 2 \Rightarrow x^2 = -1 \Rightarrow No\ tiene$$

Por simetría, el área pedida es:

$$A = 2\cdot\left(\int_0^{\frac{\sqrt{2}}{2}} (2 - (1 - x^2))\,dx + \int_{\frac{\sqrt{2}}{2}}^{\sqrt{2}} (2 - x^2)\,dx\right) = 2\cdot\left(\int_0^{\frac{\sqrt{2}}{2}} (1 + x^2)\,dx + \int_{\frac{\sqrt{2}}{2}}^{\sqrt{2}} (2 - x^2)\,dx\right) =$$

$$2\cdot\left(\left[x + \frac{x^3}{3}\right]_0^{\frac{\sqrt{2}}{2}} + \left[2x - \frac{x^3}{3}\right]_{\frac{\sqrt{2}}{2}}^{\sqrt{2}}\right) = 2\cdot\left(\frac{\sqrt{2}}{2} + \frac{\left(\frac{\sqrt{2}}{2}\right)^3}{3} + 2\sqrt{2} - \frac{(\sqrt{2})^3}{3} - \sqrt{2} + \frac{\left(\frac{\sqrt{2}}{2}\right)^3}{3}\right) =$$

$$2\cdot\left(\frac{\sqrt{2}}{2} + \frac{2\sqrt{2}}{24} + 2\sqrt{2} - \frac{2\sqrt{2}}{3} - \sqrt{2} + \frac{2\sqrt{2}}{24}\right) = 2\sqrt{2}\cdot\frac{12 + 2 + 48 - 16 - 24 + 2}{24} = 2\sqrt{2}\,u^2$$

Puntuación

1, 2, -------------- 1,25 puntos
3 ------------------ 5 “
4 ------------------ 2,5 “